DTU Library

Multivariate Modelling of Extreme Load Combinations for Wind Turbines

Dimitrov, Nikolay Krasimirov

Published in:
Proceedings of the 12th International Conference on Applications of Statistics and Probability in Civil Engineering (ICASP12)

Publication date:
2015

Document Version
Publisher's PDF, also known as Version of record

Multivariate Modelling of Extreme Load Combinations for Wind Turbines

Nikolay Dimitrov
Research Scientist, Dept. of Wind Energy, Technical University of Denmark, Denmark

ABSTRACT: We demonstrate a model for estimating the joint probability distribution of two load components acting on a wind turbine blade cross section. The model addresses the problem of modelling the probability distribution of load time histories with large periodic components by dividing the signal into a periodic part and a perturbation term, where each part has a known probability distribution. The proposed model shows good agreement with simulated data under stationary conditions, and a design load envelope based on this model is comparable to the load envelope estimated using the standard procedure for determining contemporaneous loads. By defining a joint probability distribution and full return-period contours for multiple load components, the suggested procedure gives the possibility for determining the most critical loading direction in a blade cross section, or for carrying out reliability analysis on an entire cross section.

1. INTRODUCTION

Wind turbines as moving machinery subjected to random environmental influence experience loads and deformations along multiple degrees of freedom in the structure. A fully three-dimensional stress analysis of the entire structure under dynamic loading is extremely expensive computationally. Therefore, the design loads on wind turbines are usually estimated by dynamic finite-element or modal simulations using a simplified representation of the structure in terms of beam elements. Since each node in a three-dimensional beam element has six degrees of freedom, any given load condition at a beam node, i.e., a cross section of a turbine component, will be a combination of six load components.

Due to the non-symmetric geometry of some of the turbine components such as the blades, using only the vector magnitude of the loads is not sufficient as the maximum allowable loads will vary for different load directions. In current wind turbine design guidelines (i.e., IEC 61400-1, ed.3 from 2005) the problem of finding the contemporaneous values of different load components is addressed in a relatively simple way and methods for estimating the actual joint probabilities are not available.

The present study demonstrates a multivariate model for the joint probability distribution of contemporaneous extreme loads on wind turbines. The model considers the load components as a combination of a cyclic, gravity-driven component and a randomly distributed perturbation. The subtraction of the sinusoidal signal from the random data results in better possibilities for assessing the correlations between loads in different directions, and thus leads to an improved model for the joint distribution of loads. The parameters of the marginal distributions are estimated from dynamic simulations of the wind turbine behavior under turbulent incoming wind. The joint distribution of the perturbations in different directions is obtained from the marginals using the Nataf multivariate transformation (Liu and Der Kiureghian, 1986), but can also be done with a Rosenblatt transformation (Rosenblatt, 1952), or principal component analysis (Natarajan and Velerst, 2011). A joint distribution for the loads is then obtained by convolution of the sinusoidal signal with the perturbations and integration over the rotor azimuth angles. The obvious application of this distribution model is to define the return period contours for extreme load combinations and thus the design load envelope, but it also gives the possibility for carrying out reliability analysis of wind turbine components.

2. EXTREME LOADS FROM AEROELASTIC SIMULATIONS

2.1. Input description

The limit state considered in this study is the multi-axial extreme load acting on a cross section from a wind turbine blade. The wind turbine used as an example is the DTU 10MW reference wind turbine (Bak et al., 2013) with rotor diameter of 179m and hub height of 119m. The loads are determined from aeroelastic simulations using the Hawc2 code (Larsen and Hansen, 2012). Fitting a representative probabilistic model of the loads is only feasible under stationary conditions, i.e., constant mean wind speed, turbulence, etc., and if long-term load distributions under varying conditions are required, they are estimated by fitting a number of short-term distributions under stationary conditions, followed by integration over the respective distribution of long-term variables (see e.g. the IEC61400-1, ed.3 standard, 2005). For simplicity we consider stationary time series with a fixed mean wind speed of 11m/s, turbulence intensity 0.176 (IEC turbulence class B), and wind shear exponent of 0.2. The loads on a blade cross section are also influenced by the blade pitch angle, where an increase in the pitch angle normally leads to a reduction in the aerodynamic thrust acting on the rotor, and hence to reduction in the flapwise loads. We choose to take into account the time periods in the simulations when the blade pitch angle is nearly zero (less than 0.5°), which can be considered as a worst-case scenario since the flapwise loads are highest at zero pitch angle and 11m/s is the wind speed at which the highest extreme loads are typically attained. This set of conditions is sufficient for demonstrating the methods for modelling of the joint distribution of extreme loads. However, for design purposes it will be relevant to fit a number of short-term distributions under different wind conditions and taking account of different blade pitch angles, and then estimate the design load envelope by integrating the short-term distributions into a long-term distribution.

2.2. Extraction of load extremes

The ultimate limit state design loads for a wind turbine are specified as the extreme loads with a given long period of recurrence (i.e. 1 year or 50 years). Estimating the distribution of the extremes requires extracting independent peaks from the time series under consideration. In the multivariate case, the definition of an extreme event is more complicated as all related load components have to be taken into consideration. We adopt an approach where a point in time $t = \tau$ is considered a load peak if it satisfies the following two conditions:

1) There is a load reversal of the flapwise load M_x at $t = \tau$, i.e,
$$|M_x(\tau)| > |M_x(\tau - \Delta t)| \text{ and}$$
$$|M_x(\tau)| > |M_x(\tau + \Delta t)|$$
2) There is no larger peak within 1s before and after $t = \tau$:
$$\max_{M_x(t)}(t \in [\tau - 1s, \tau + 1s]) = M_x(\tau).$$

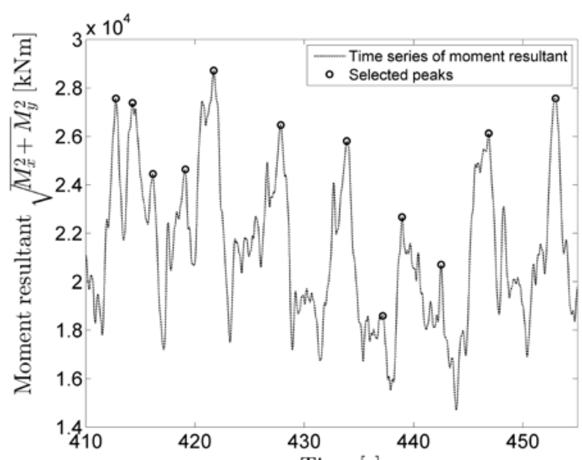

Figure 1: Example of load peaks extracted from simulated time history for blade bending moments M_x and M_y.

Figure 1 shows an example of the time series for the resultant of the flapwise and edgewise bending moments $M_R = \sqrt{M_x^2 + M_y^2}$ and the peaks identified using the criterion given above.

When a probability distribution is fit to the load peaks extracted with the above procedure, the probability values will be relative to the number of peaks used to fit the distribution. This is a so-called local distribution of peaks. In order to determine the actual short-term distribution of peaks with respect to a specific reference period, T, the local distribution has to be raised to the power $n(T)$, where n is the expected number of load peaks within one time period T:

$$F_{short-term}(M|\mathbf{X},T) = \left(F_{local}(M|X)\right)^{n(T)} \quad (1)$$

For example, if the desired reference time period is $T = 10min$ (the typical duration of a single aeroelastic simulation) , $n(T)$ equals the number of load peaks extracted per time series.

3. JOINT PROBABILITY DISTRIBUTION OF MULTI-AXIAL LOADS

For the purpose of detailed reliability analysis or structural design of a wind turbine blade section in general, we need the probability of observing a given combination of load components acting on the cross section. The components are typically given in six degrees of freedom; that is moments and forces acting in three axes of a Cartesian coordinate system. The most critical of these six components are the transverse bending moments M_x and M_y (flapwise and edgewise bending moments). The other four components can either be sufficiently represented as functions of M_x and M_y, or their magnitude is sufficiently small as to not influence the design significantly. Therefore, the major part of the problem of determining the probability of given load combination amounts to finding the joint distribution of the extreme flapwise and edgewise loads, $f_{M_x M_y}$:

$$\begin{aligned} f_{M_x M_y} &= f_{M_x}(M_x) f_{M_y}\left(M_y \middle| M_x\right) \\ &= f_{M_y}\left(M_y\right) f_{M_x}\left(M_x \middle| M_y\right) \end{aligned} \quad (2)$$

Figure 2 shows an example of the joint probability density of the extremes of M_x and M_y for a typical cross section in a wind turbine blade.

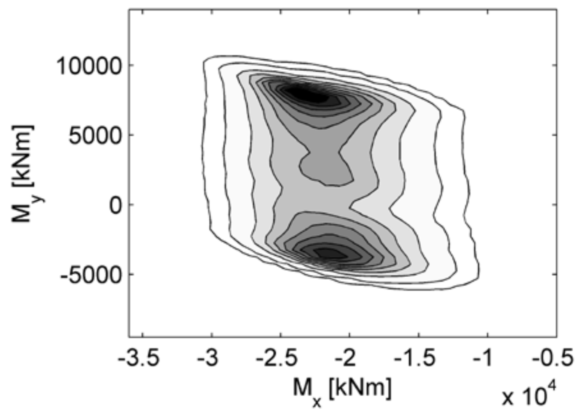

Figure 2: Joint pdf of flapwise bending moment M_x and edgewise bending moment M_y for a wind turbine blade cross section. Based on 100 Hawc2 simulations for a 10MW turbine at 11m/s wind speed, section located 46.6m from the blade root.

3.1. Marginal distributions of load components

The edgewise loads are mainly gravity-driven and as such have a strong sinusoidal component caused by the rotation of the turbine. As a result, the marginal distribution of M_y is bimodal (Figure 3). The presence of the sinusoidal signal means that M_y can be considered as the sum of a sinusoidal function S_y with amplitude a and phase φ, and a random “noise” component ϵ_y:

$$\begin{aligned} M_y &= \mu_y + a \cdot \sin(\omega t + \varphi) + \epsilon_y \\ &= S_y + \epsilon_y \end{aligned} \quad (3)$$

where μ_y is the mean moment, ω is the angular velocity, t is the time, and $\omega t = \gamma$ is the rotor azimuth. The sinusoidal signal considered as a random variable can be represented either as a sinewave-function of a uniformly-distributed azimuth variable, or as a standalone random variable using the arcsine distribution:

$$f_{S_y}(S_y) = \frac{1}{\pi\sqrt{a^2 - \left(S_y - \mu_y\right)^2}} \qquad (4)$$

The perturbation term ϵ_y can be represented by any suitable statistical distribution, and in the case when the sinusoidal component is much bigger than the noise component, a normal distribution for ϵ_y seems to be a good choice. The probability distribution of the edgewise moment M_y is then considered as the distribution of the sum of two independent random variables, given by the convolution of the marginal distribution functions of S_y and ϵ_y:

$$f_{S_y+\epsilon_y}(M_y) = \int_{-\infty}^{\infty} f_{S_y}(\tau) f_{\epsilon_y}\left(M_y - \tau\right) d\tau \qquad (5)$$

An example fit of the distribution model from equation (4) to data from aeroelastic simulations is shown on Figure 3.

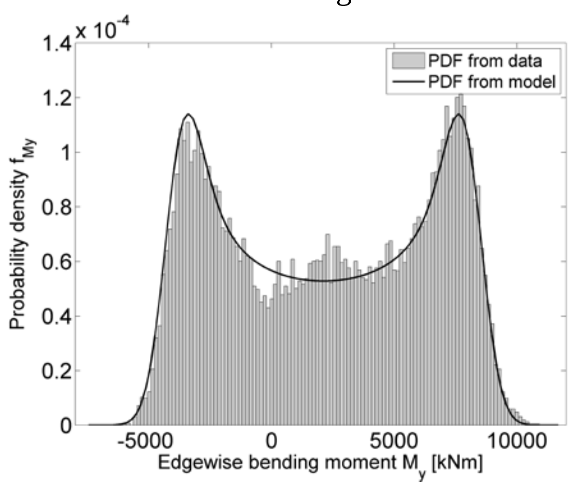

Figure 3: Marginal probability density function of the extreme edgewise bending moment M_y: comparison between simulations and a distribution model based on convolution of a sinusoidal function and a normal distribution.

The flapwise bending moment is mainly driven by turbulence, however there is also a small sinusoidal component due to the blade pitch axis not being exactly in the rotor plane. As a result, the distribution of M_x can also be modelled as the convolution of a sinusoidal distribution and a perturbation, where the perturbation term will have the dominating contribution, and in this case is modelled by the Generalized Extreme Value (GEV) distribution. Figure 4 compares the fitted distribution for M_x to the empirical distribution from observations. Since for the flapwise load we expect that the distribution tails have higher importance than the values close to the mean, the comparison is shown in terms of the cumulative distribution function $F_{M_x}(M_x)$ and its complement: $P_{M_x}(M_x) = 1 - F_{M_x}(M_x)$.

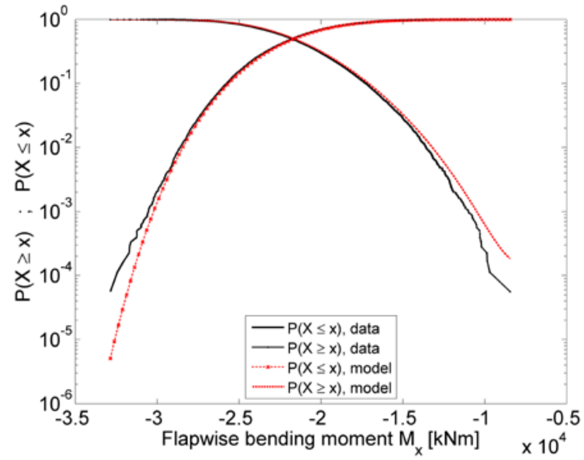

Figure 4: Marginal distribution function of the extreme flapwise bending moment M_x: comparison between simulations and a distribution model based on convolution of a sinusoidal function and Generalized Extreme Value distribution.

In the present case the GEV distribution fits reasonably well both the higher and lower tails of the marginal distribution of the flapwise moment, M_x. However, this is not necessarily always true, and in some other cases more advanced fitting may be required, e.g. separate fits to the upper and lower tails of the distribution, or other approaches.

3.2. Modelling of the joint distribution of flapwise and edgewise blade moments

Several methods for obtaining design load combinations based on the time series for multiple load components have been employed in previous studies and in design codes:

- The IEC 61400-1 standard suggests a simple technique where the load combination case combines the highest extreme flapwise load with the mean edgewise load, or in a more conservative manner the combination can also consist of the extremes for both flapwise and edgewise loads.
- The joint distribution of M_x and M_y is expressed in terms of the marginal distributions of uncorrelated standard variables by the use of a multivariate transformation such as the Nataf transform, the Rosenblatt transformation, or Principal Component Analysis (see e.g. Natarajan et al.).

Obviously, the procedure recommended in the IEC 61400-1 standard is simple, however not necessarily providing the designer with the right information. The methods employing multivariate transformations also have limitations: the Rosenblatt transformation requires fitting of a number of conditional distributions, which limits its usability for very high load values where the number of observations is small; and the Nataf transform and Principal Component Analysis require the use of the correlation matrix with Pearson's correlation coefficients, which are only representative for data with linear correlation. As seen on Figure 1, this is not the case for M_x and M_y. Figure 5 demonstrates the limitations of this approach, showing a comparison of the joint PDF of M_x and M_y observed from simulations to the joint PDF obtained by using a Nataf transformation (Liu & Der Kiureghian, 1998):

$$f_{M_xM_y}(M_x, M_y) = f_{M_x}(M_x) f_{M_y}(M_y) \frac{\varphi_{\mathbf{n}}(\mathbf{z}, \mathbf{R}')}{\varphi(z_1)\varphi(z_2)} \quad (6)$$

where $z_i = \Phi^{-1}\left[F_{M_i}(M_i)\right]$, $i = x, y$, and $\varphi_{\mathbf{n}}(\mathbf{z}, \mathbf{R}')$ is the n-dimensional normal PDF for variables with zero mean, unit variance and correlation matrix $\mathbf{R}'$. The correlation coefficients ρ'_{ij} in $\mathbf{R}'$ are function of the correlation coefficients of the data, ρ_{ij}, and can be determined by an iterative solution or, for some distributions, calculated from tables, see Liu & Der Kiureghian for details. The marginal distributions of M_x and M_y are calculated based on the convolution rule as defined in Equation (5): $f_{M_x} = f_{S_x+\epsilon_x}$ and $f_{M_y} = f_{S_y+\epsilon_y}$.

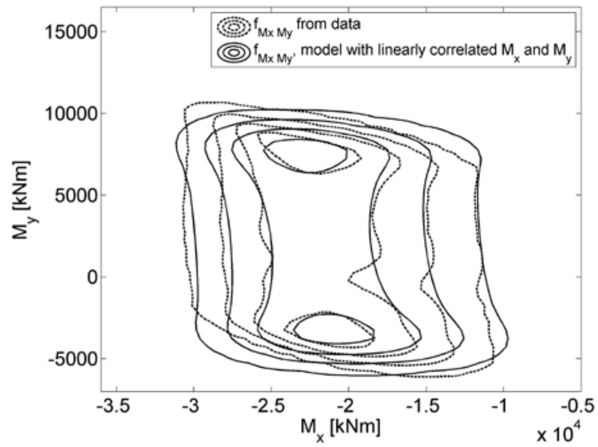

Figure 5: Comparison of the joint probability density for blade flapwise and edgewise extreme bending moments, estimated from data and by directly using a multivariate transformation on the marginal distributions of M_x and M_y.

The transformation used to create Figure 5 requires the use of the correlation coefficient between M_x and M_y, which according to the data is $\rho_{MxMy} = -0.24$. However, as mentioned earlier Pearson's correlation coefficient describes well only linearly dependent variables, which is not the case for the present problem – and subsequently the multivariate distribution based on this approach does not fit the data well. The nonlinearity in our case is caused by the bi-modal behavior of the loads due to the cyclic components in them, and once the sinusoidal part is eliminated, the dependence between the perturbation terms ϵ_x and ϵ_y is well described by a linear correlation coefficient. As a result, the joint distribution $f_{\epsilon_x\epsilon_y}$ can be modelled adequately by with Equation 6, as Figure 6 shows. For comparison, the correlation coefficient $\rho_{\epsilon_x\epsilon_y}$ is -0.69, significantly larger than ρ_{MxMy}.

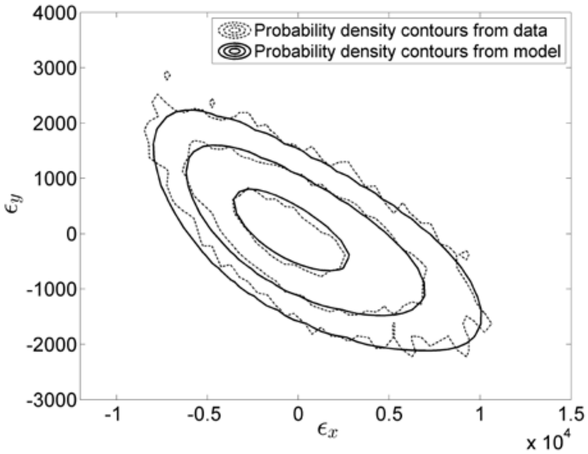

Figure 6: Joint probability density function of the noise terms ϵ_x and ϵ_y. Comparison between simulations and a multivariate distribution fit.

Based on this observation and knowing that the bending moments M_x and M_y can be expressed as function of the perturbation terms ϵ_x, ϵ_y and the rotor azimuth γ, we propose that the joint distribution $f_{M_x M_y}$ can be modelled using the joint distribution of ϵ_x, ϵ_y and γ:

$$
\begin{aligned}
&f_{M_x M_y}(M_x, M_y) = \\
&= \int_0^{2\pi} f_\gamma(\gamma) f_{\epsilon_y}(\epsilon_y|\gamma) f_{\epsilon_x}(\epsilon_x|\gamma, \epsilon_y)\, d\gamma \\
&= \int_0^{2\pi} f_\gamma(\gamma) f_{\epsilon_y}\left(\epsilon_y \middle| S_y(\gamma, M_y)\right) \\
&\cdot f_{\epsilon_x}\left(\epsilon_x \middle| S_x(\gamma, M_x), \epsilon_y\right) d\gamma
\end{aligned}
\tag{7}
$$

where

$$
\begin{aligned}
f_{\epsilon_x \epsilon_y} &= \int_0^{2\pi} f_{\gamma \epsilon_x \epsilon_y}(\gamma, \epsilon_x, \epsilon_y) d\gamma \\
&= \int_0^{2\pi} f_\gamma(\gamma) f_{\epsilon_y}(\epsilon_y|\gamma) f_{\epsilon_x}(\epsilon_x|\gamma, \epsilon_y) d\gamma
\end{aligned}
\tag{8}
$$

This approach results in a significantly improved match to the joint probability distribution of M_x and M_y, which is shown in Figure 7.

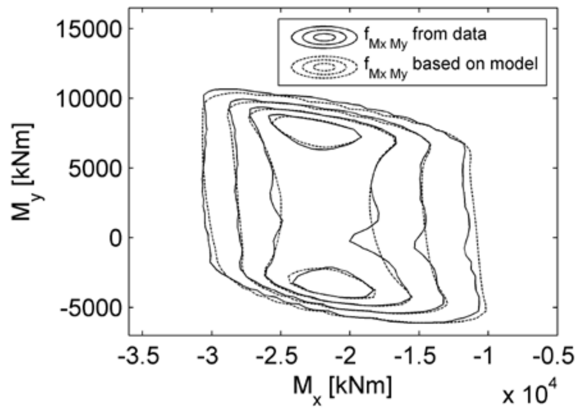

Figure 7: Comparison of the joint probability density for blade flapwise and edgewise extreme bending moments, estimated from data and by using a multivariate model based on the marginal distributions of perturbation terms ϵ_x and ϵ_y and integrated over the rotor azimuth γ.

3.3. Return period contours for the joint distribution of M_x and M_y

Return period contours are evaluated using the inverse-FORM (i-FORM) procedure, Winterstein et al. (1993). The i-FORM calculations are carried out in terms of standard normal, uncorrelated variables, meaning that for using the method, the distributions of M_x and M_y have to undergo two transformations: 1) transformation to a pair of dependent standard normal variables U_1 and U_2, and 2) to account for dependency and transform U_1 and U_2 into a pair of independent standard variables, Y_1 and Y_2. The latter transformation is done using Cholesky decomposition of the correlation matrix $\mathbf{R}'$ (derived from the correlation matrix of the data $\mathbf{R}$ via the Nataf transformation), obtaining a transformation matrix $\mathbf{T}$ where $\mathbf{TT}' = \mathbf{R}'$, and the final result is found by $\mathbf{Y} = \mathbf{TU}$.

In order to verify the above calculation, a large number of time series under the same conditions described in Section 2 were simulated for obtaining a reference. Based on a total of 30000 time series (approximately equal to 7 months) under stationary conditions, load combination contours corresponding to 1-day and 1-month return periods are estimated. Figure 8 shows the comparison between the reference simulations and the joint distribution model. The agreement between the model and the reference data is good in general, with the exception of a slight deviation in the region of maximal flapwise load. Considering that the model is based on just 100 time series compared to the 30000 used for the reference calculation, at least part of the disagreement can be attributed to statistical uncertainty.

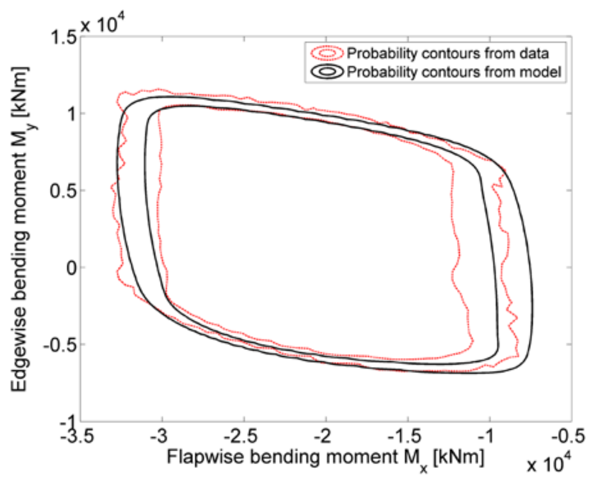

Figure 8: Probability contours corresponding to 1-day and 1-month return periods, estimated with the multivariate extreme load model, compared to contours estimated from 30000 time series.

3.4. Comparison to contemporaneous loads calculation

The IEC 61400-1 standard, ed.3 first amendment (2010) recommends a simplified procedure for obtaining contemporaneous loads, where the characteristic extreme value for a given load component is found as the mean of the maxima from each realization (e.g. turbulence seed), and the contemporaneous load for other components is found as the mean of the contemporaneous values from each realization. The design load is then calculated by multiplying the characteristic loads by a safety factor of 1.35. Considering that extreme design load cases in the IEC61400-1 are normally tuned to represent events with 50-years recurrence periods, on Figure 9 we compare the contemporaneous design loads obtained using the IEC-procedure to the 50-year return period contour estimated using the proposed joint distribution model. As the figure shows, the contemporaneous design loads based on the standard procedure agree with the 50-year joint probability contour, but with a reduced safety factor of 1.25 instead of the standard 1.35. This observation however can only be verified for the simulation conditions used for the present study. For other conditions (e.g. load cases with extreme turbulence as DLC1.3 which are often design-driving for blades), the standard contemporaneous loads and the joint probability distribution model might compare differently.

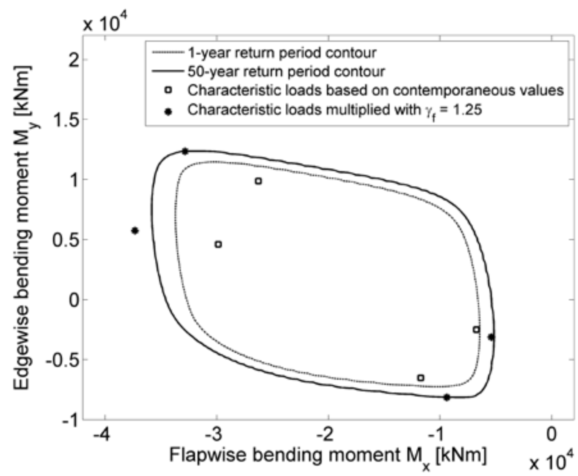

Figure 9: Comparison of the 1-year and 50-year return period contours estimated by the multivariate model to the design envelope determined using the standard contemporaneous loads procedure. The value of the safety factor used is tuned so the loads multiplied with it coincide with the 50-year contour.

4. DISCUSSION AND CONCLUSIONS

In the present paper, we demonstrated a model for estimating the joint probability distribution of two wind turbine blade load components. The study addressed the problem of modelling the probability distribution of load time histories with large periodic components by dividing the signal into a periodic part and a perturbation term, where each part has a known probability distribution. The proposed model shows good agreement with simulated data under stationary conditions, and a design load envelope based on this model is comparable to the load envelope estimated using the standard procedure for determining contemporaneous loads. By defining a joint probability distribution and full return-period contours for multiple load components, the suggested procedure gives the possibility for determining the most critical loading direction in a blade cross section, or for carrying out reliability analysis on an entire cross section. This makes the model a potentially valuable tool to wind turbine designers as it provides additional information compared to what is learned by assessing just a few selected loading directions.

ACKNOWLEDGEMENTS
The work presented herein is a part of the Danish Energy Technology Development and Demonstration (EUDP) project titled, "Demonstration of a basis for tall wind turbine design", Project no 64011-0352. The financial support is greatly appreciated.

REFERENCES

Liu, P. L and Der Kiureghian, A. (1986) "Multivariate distribution models with prescribed marginals and covariances" *Probabilistic Engineering Mechanics*, 1(2), 105-112

Rosenblatt, M. (1952) "Remarks on a multivariate transformation" *Annals of Mathematical Statistics*, 23, 470-472

Natarajan, A. and Velerst, D. R. (2011) "Outlier robustness for wind turbine extrapolated extreme loads", *Wind Energy*, DOI 10.1002/we.497

Bak, C., Zahle, F., Bitsche, R., Kim, T., Yde, A., Henriksen, L.C., Natarajan, A., Hansen, M. (2013) "*Description of the DTU 10 MW reference wind turbine*", DTU Wind Energy Report-I-0092

Larsen, T. J., Hansen, A. M. (2012) "*How to HAWC2, the user's manual*." Tech. Rep. Risø-R-1597(ver.4-3) (EN), DTU Wind Energy, Roskilde, Denmark, April 2012

IEC. (2005) "*International Standard IEC 61400-1: Wind Turbines – Part 1: Design Guidelines*"

Winterstein, S. R., Ude, T. C., Cornell, C. A., Bjerager, P., Haver, S. (1993) "Environmental parameters for extreme response: inverse FORM with omission factors", *in Proceedings of the ICOSSAR-93 conference, Innsbruck, Austria.*

IEC. (2010) "*International Standard IEC61400-1, Ammendment 1. Wind Turbines - Part 1: Design Guidelines*"